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Breaking elongation distributions of single fibres

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The breaking elongation distributions of cotton, regenerated cellulose and polyester fibres have been investigated. The distributions of the cotton fibres are positively skewed whereas those of the regenerated cellulose fibres are negatively skewed. All of these distributions have longer tails than normal distributions. Although being approximately symmetric, the distributions of the polyester fibres also show longer tails than normal distributions. A mixture of Gaussian and Lorenzian functions is suggested for the resolutions of the symmetric distributions. For the asymmetric distributions with positive skewness, Gamma functions provide the best fit for their distributions. A mixture of Normal and Gumbel (largest or smallest) can also provide close curve-fittings to the asymmetric distributions with negative skewness as well as positive skewness.

1. Introduction

Fibres are the most elementary constituents in yarn, fabric and composite structures. Their tensile properties are thus among the most important physical parameters from which the properties of their assemblies are determined. Measurements of single fibre tensile properties are time-consuming and impractical for large scale and industrial applications. For cotton fibres, methods involving measurements of fibre bundles have been developed. The Stelometer, Pressley, Fafegraph-M and the high volume instrument (HVI) are all examples of bundle fibre testers [1]. The tensile properties of single cotton fibres are then estimated from an established correlation between the bundle fibres and single fibres [2–7].

For a basic understanding of fibre tensile properties, however, direct measurements of single fibres are crucial. This is due to the scattering nature of single fibre tensile properties which are described not only by arithmetic mean values, but also by statistical distribution functions [8]. The distribution functions of single fibre tensile properties, in particular strength and modulus, have been analysed [9–14]. Much less is known about the fibre elongation distribution. A very early work assumed the elongation distributions of fibres to be normally distributed [15]. A more recent report on cotton fibres has also shown that single fibre breaking elongation fits normal distribution [16]. However, others have indicated that the distributions of cotton fibre breaking elongation have a skewed nature [3, 6, 17]. Rayleigh distribution has been deemed suitable for cotton fibre elongation [3]. Furthermore, a recent study has suggested possible significance of the standard deviation of the single fibre breaking elongation on the strength of fibre bundles [18]. Thus the opposing results on the distributions of single fibre breaking elongation require further verification.

The aim of this work is to analyse the distributions of single fibre breaking elongation with more in-depth statistical analysis. Several fibre types, i.e. three cottons, two regenerated cellulosic, and two polyester fibres, are included. Measurements of single fibre tensile properties have been performed with the aid of a high-speed and automated single fibre tensile tester [16]. In this manner, a sufficiently large data base can be established for a meaningful statistical analysis.

2. Experimental details

2.1. Single fibre tensile measurement

A Mantis single fibre tensile tester (Zellweger Usters, Inc.) was used in this work to accumulate a large number of data. The Mantis instrument does not require mounting preparation of single fibres. The operations of alignment, straightening and gripping of each single fibre are automatic and quick. Tensile measurements are performed at a 3.2 mm gauge length using an internal pressure transducer at a 1.0 mm s^{-1} strain rate. The force transducer calibration is calibrated using five standard weights ranging from 1.526 to 19.599 g. In addition, fibre diameters or ribbon widths are determined by an optical sensor. The fibre is pretensioned under a small stress (typically 0.5 g) or small extension (typically 0.5 mm) whichever is reached first, at a fixed length of 10 mm. The attenuation of the light gives the fibre diameter or ribbon width. All measurements were performed at a constant condition of 70 °F and 65% relative humidity.

2.2. Materials

Three types of cotton fibres, two types of regenerated cellulose fibres and two polyester filaments were measured. The three cottons were a micronaire standard,

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fibres from a carded silver (Nisshinbo Inc.), and a Mantis calibration standard (Zellweger Usters, Inc.). For ease of designation, these three cotton fibres are referred to as Cotton I, II, III, respectively. The two polyester fibres studied are the Mantis calibration standard fibres (Zellweger Uster, Inc.) and drawn and crimped fibres (du Pont de Nemours, Inc.). They are referred to as Polyester I and II, respectively. The Mantis calibration standard rayon fibres (Zellweger Uster, Inc.) and Lyocell filaments (Courtaults Inc.) are also included. All samples were conditioned at 70 °F and 65% relative humidity for at least 48 h prior to the measurements.

2.3. Sample sizes

For studying the fibre elongation distribution, the minimum sample sizes should be determined by the estimated population mean value and the standard deviation. The minimum sample sizes for estimating the mean ($n_{\bar{x}}$) and the standard deviation (n_s) of a normally distributed population with given accuracy d and confidential coefficient α are [19]:

$$n_{\bar{x}} = \left(\frac{cv \times z_{\alpha/2}}{d_{\bar{x}}}\right)^2 \tag{1}$$

$$n_s \approx 1 + 0.5(z_{\alpha/2}/d_s)^2 \tag{2}$$

where $d_{\bar{x}} = (\bar{x} - \mu)/\mu$ and $d_s = (s - \sigma)/\sigma$. Since σ is generally unknown, sample standard deviation s is usually used. For the cotton fibres, the coefficient of variation (cv) is around 35% [4]. The minimum sample sizes for $n_{\bar{x}}$ and n_s are calculated to be 300 when $d_{\bar{x}} = 0.04$, $d_s = 0.08$, and $\alpha = 95\%$. The actual sample sizes in this work are listed in Table I.

3. Results and discussion

3.1. Single fibre breaking elongation

3.1.1. Mean values and standard deviations

The extreme values, mean values, standard deviations and coefficient of variations of the breaking elongation data of the seven fibres are summarized in Table I. The mean elongation values and standard deviations of the polyester fibres are the highest, whereas those of the cotton fibres are the lowest. The coefficient of variation (cv) , however, is the best parameter for comparing the variances among these fibres. The cotton fibres have the highest cv values (33–36%) whereas rayon and Lyocell fibres have the lowest cv values (17–19%).

TABLE I Single fibre breaking elongation data

Sample	n	x_{min} (%)	x_{max} (%)	$\bar{x}$ (%)	s	cv (%)
Cotton I	997	0.1	29.0	12.6	4.17	33.2
Cotton II	784	2.6	23.7	9.9	3.33	33.5
Cotton III	497	1.6	26.2	10.4	3.76	36.3
Rayon	528	1.5	41.0	24.0	4.48	18.7
Lyocell	290	5.5	30.1	20.1	3.44	17.2
Polyester I	613	26.8	104.1	63.6	13.90	21.8
Polyester II	340	31.6	124.6	78.2	14.96	19.1

3.1.2. Elongation histograms

Any histogram should be constructed with an optimized number of intervals. For the single fibre elongation data, the rule of using the square root of sample size for setting the number of intervals would give a large number of small intervals. Too many intervals may lead to many empty intervals as well as widely different frequencies in the adjacent intervals; whereas too few intervals would not show the variability in the data [20]. The single fibre breaking elongation histograms have been constructed using 12–18 intervals from the minimal to maximal elongation values.

The elongation histograms of all seven fibres appear to be less symmetric, and have longer tails than a normal distribution (Figs 1–3). The statistical measurements for symmetry and long-tails are skewness (g_1)

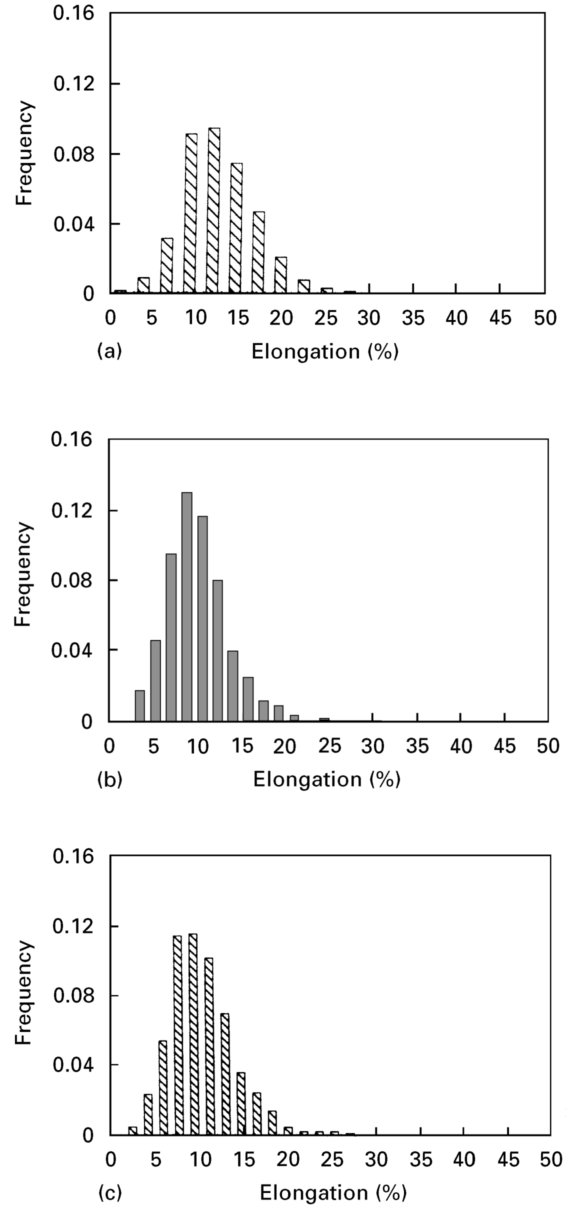

Figure 1 Elongation histograms of the cotton fibres: (a) cotton I, (b) cotton II and (c) cotton III.

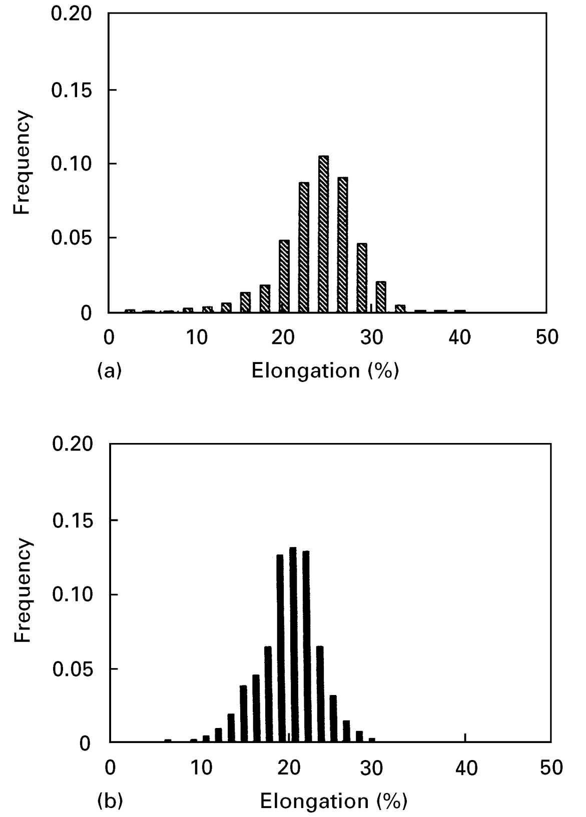

Figure 2 Elongation histograms of the regenerated cellulose fibres: (a) rayon and (b) Lyocell.

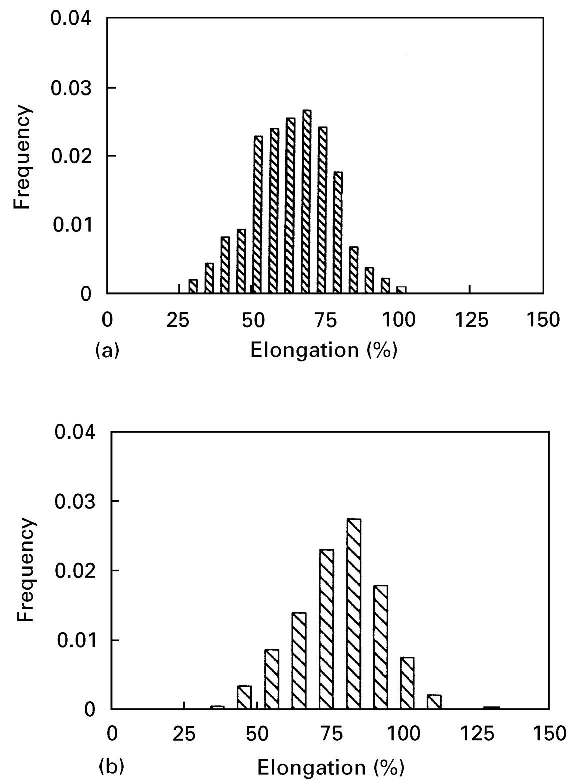

Figure 3 Elongation histograms of the polyester fibres: (a) polyester I and (b) polyester II.

and kurtosis (g_2), respectively [21]:

$$g_1 = \sum (x_i - \bar{x})^3/(Ns^3) \quad (3)$$

$$g_1 = \sum (x_i - \bar{x})^4/(Ns^4) - 3 \quad (4)$$

TABLE II The skewness (g_1) and kurtosis (g_2) of the obtained data, and the corresponding standard errors (g_{1c} and g_{2c}) of a normal distribution

Sample	g_1	g_2	g_{1c}	g_{2c}
Cotton I	0.53	0.44	0.08	0.16
Cotton II	0.70	0.79	0.09	0.20
Cotton III	0.90	1.60	0.11	0.22
Rayon	− 0.92	3.18	0.11	0.21
Lyocell	− 0.56	1.10	0.14	0.29
Polyester I	− 0.05	− 0.25	0.10	0.20
Polyester II	− 0.12	− 0.01	0.13	0.27

The standard errors of g_1 and g_2 for a normal distribution are $(6/N)^{1/2}$ and $(24/N)^{1/2}$, respectively. Asymmetry is shown by a skewness value larger than the corresponding standard error, while a positive value indicates a longer tail on the right and a negative value indicates a longer tail on the left. A kurtosis value which is significantly greater than the corresponding standard error indicates a distribution that has longer-tails than a normal distribution. Table II summarizes the skewness (g_1) and kurtosis (g_2) values of the fibres.

The histograms of all three cotton fibres have larger skewness values than the standard errors, thus indicating asymmetric elongation frequencies. The positive skewness values of these distributions also indicate longer tails on the right side of the distribution. The longer right tail has the physical meaning of a larger amount of cotton fibres possessing lower elongations than their mean values. Also, small amounts of cotton fibres with greater elongations than the mean value exist and govern the distribution shapes. Moreover, the significantly higher kurtosis values than the standard errors of normal distributions for the three cotton fibres also show that they have longer tails than normal distributions. Hence, the elongations of the three cotton fibres should by no means be considered as normal distributions. Observations of asymmetric elongation distributions have been reported by others [3, 6, 17]. However, no detailed statistical analysis was reported in those works. The asymmetric distributions shown by these three cotton populations do not support Cui's [16] claim that the cotton elongation distributions obey normal distributions.

The fact that larger amounts of fibres in these three cotton populations possess lower elongation than the mean values has significant implications in the strength of cotton assemblies, such as bundles, yarns, or fabrics. The lowest extensible fibres break first when the fibre assemblies are stretched and thus vitally affect their ultimate strength. The sources of these less extensible fibres are not clear. They may be less developed fibres and/or those damaged by ginning and/or other processing. Better understanding of the reasons for the larger proportion of cotton fibres with low breaking elongation is one of the most important and fundamental aspects in the effort to improve the strength of cotton assemblies.

In contrast, the regenerated celluloses (rayon and Lyocell) show negative skewness which are also significantly larger than the standard errors, confirming asymmetric with longer tails on the left side of the distributions. These asymmetric distributions can be physically interpreted as there are proportionally more fibres with higher elongations, but there are also some fibres possessing very much lower elongations than the mean values. The kurtosis values of the two regenerated cellulose fibres are significantly larger than the standard errors, suggesting larger amounts of extreme fibre elongations or longer tails than the normal distribution. Therefore, the elongation distributions of these two regenerated celluloses should not be considered as normal distributions either.

The skewness values for the breaking elongations of the two polyester fibres do not significantly differ from their standard errors. Therefore, their distributions are approximately symmetric. The kurtosis value of polyester I is not significantly different from zero, suggesting that the fibres are from a normally distributed population. The kurtosis value of polyester II is larger than the standard error, suggesting that the symmetric distribution has long tails on both sides.

3.2. Determination of the distribution function

3.2.1. Symmetric distributions

The elongation frequency distributions of the two polyester fibres are approximately symmetric, and therefore can be fitted by mathematically symmetric functions. Although the elongation of polyester I can be considered as being from a normal distribution, the elongation of polyester II has a longer tail than a normal distribution. For the best curve-fitting, we propose to use a mixture of Gaussian and Lorentzian functions to resolve the sample frequency distribution in terms of maximum amplitude (A), width at the half maximum amplitude (W), mean value at the maximum amplitude position (P) and profile function parameter (f)

$$G_{(A,W,P)} = A \exp\left[-\frac{4(X-P)^2}{W^2}\right] \tag{5}$$

$$L_{(A,W,P)} = \frac{A}{1+\dfrac{4(X-P)^2}{W^2}} \tag{6}$$

where $G_{(A,\,W,\,P)}$ and $L_{(A,\,W,\,P)}$ are the Gaussian and the Lorentzian functions, respectively. The sample frequency distribution is then defined as:

$$p(x) = (1-f)\times G_{(A,\,W,\,P)} + f \times L_{(A,\,W,\,P)} \tag{7}$$

where f is a profile function parameter which defines the form of the frequency distribution. The f is set in the range of $0 \leqslant f \leqslant 1$, where f is 1 for the pure Lorentzian function and 0 for the pure Gaussian function. This evaluation method is well established in the analyses of X-ray, infrared and other spectroscopy [22].

3.2.2. Asymmetric distributions

For asymmetric distributions of single fibre breaking elongations, the statistical asymmetric functions, such as Weibull, Log-normal, Gamma, and Beta were employed for curve resolutions. The probability density functions (pdf) of the above functions and their corresponding expected values ($E(x)$) and variance ($D(x)$) are summarized in Table III. The corresponding parameters of each function were calculated by the numerical method of maximum likelihood estimation using Biomedical (BMDP) statistical computer programs [21]. For Gamma and Beta distributions, the unknown parameters were estimated by the moment methods, namely, the sample mean value and the variance being set equal to the corresponding expressions for the population.

TABLE III Employed asymmetric standard statistical functions and their expected values and variances

Function	pdf	$E(x)$	$D(x)$
Log-normal $\ln(c, m^2)$	$p_{\ln}(x) = \begin{cases} \dfrac{1}{(2\pi)^{1/2}mx}\exp\left[-\dfrac{(\ln x - c)^2}{2m^2}\right], & x>0 \\ 0, & x \leqslant 0\end{cases}$ $-\infty < c < \infty,\quad m>0$	$\exp\left(c+\dfrac{m^2}{2}\right)$	$exp\,(2c+m^2)\,[\exp(m^2)-1]$
Weibull $W(c, \alpha, m)$	$p_{\mathrm{w}}(x) = \begin{cases} \dfrac{m}{\alpha}(x-c)^{m-1}\exp\left[-\dfrac{(x-c)^m}{\alpha}\right], & x \geqslant c \\ 0, & x<c\end{cases}$ $m>0,\quad \alpha>0$	$\alpha^{1/m}\Gamma\left(1+\dfrac{1}{m}\right)+c$	$\alpha^{1/m}\left[\Gamma\left(1+\dfrac{2}{m}\right) - \Gamma^2\left(1+\dfrac{1}{m}\right)\right]$
Gamma $\Gamma(c, \alpha, m)$	$p_{\Gamma}(x) = \begin{cases} \dfrac{\alpha^m}{\Gamma(m)}(x-c)^{m-1}\exp[-\alpha(x-c)], & x>c \\ 0, & x \leqslant c\end{cases}$ $\alpha>0,\quad m>0$	$\dfrac{m}{\alpha}+c$	$\dfrac{m}{\alpha^2}$
Beta $\beta(m, n)$	$p_{\beta}(x) = \begin{cases} \dfrac{(x-c)^{m-1}\times(c+\alpha-x)^{n-1}}{\alpha^{m+n-1}\times B(m,n)}, & c \leqslant x \leqslant (c+\alpha) \\ 0, \quad x<c,\ x>(c+\alpha) & \end{cases}$ $m>0,\quad n>0$	$\dfrac{\alpha\times m}{m+n}+c$	$\dfrac{\alpha^2\times m\times n}{(m+n)^2(m+n+1)}$

Additionally, we propose using a mixture of normal and Gumbel (largest and smallest) distribution for the asymmetric curve-fitting.

$$p(x) = \frac{(1-f)}{(2\pi\sigma_1^2)^{1/2}} \exp\left[\frac{(x-\mu_1)^2}{2\sigma_1^2}\right] + \frac{f}{\sigma_2} \exp\left\{\frac{1}{\sigma_2}(x-\mu_2) - \exp\left[-\frac{1}{\sigma_2}(x-\mu_2)\right]\right\} \quad (8)$$

$$p(x) = \frac{(1-f)}{(2\pi\sigma_1^2)^{1/2}} \exp\left[\frac{(x-\mu_1)^2}{2\sigma_1^2}\right] + \frac{f}{\sigma_2} \exp\left\{\frac{1}{\sigma_2}(x-\mu_2) - \exp\left[-\frac{1}{\sigma_2}(x-\mu_2)\right]\right\} \quad (9)$$

where f is a profile function parameter which is set in the range of $0 \leqslant f \leqslant 1$. f is 1 for the pure Gumbel and 0 for the pure normal distribution functions. The mixture distribution function usually has five unknown parameters, $\mu_1, \sigma_1, \mu_2, \sigma_2, f$. Again, these unknown parameters were estimated by the maximum likelihood estimation using the BMDP programs.

3.3. Resolution and parameter estimation
3.3.1. Symmetric distributions

The elongation distributions of the polyester fibres I and II are closely fitted using the mixture of Gaussian and the Lorentzian functions (Fig. 4). The resolved parameters are summarized in Table IV. It can be seen that the elongation distribution of polyester I fibres is approximately a normal population while the elongation distribution of polyester II fibres includes nearly 7% Lorenzian content.

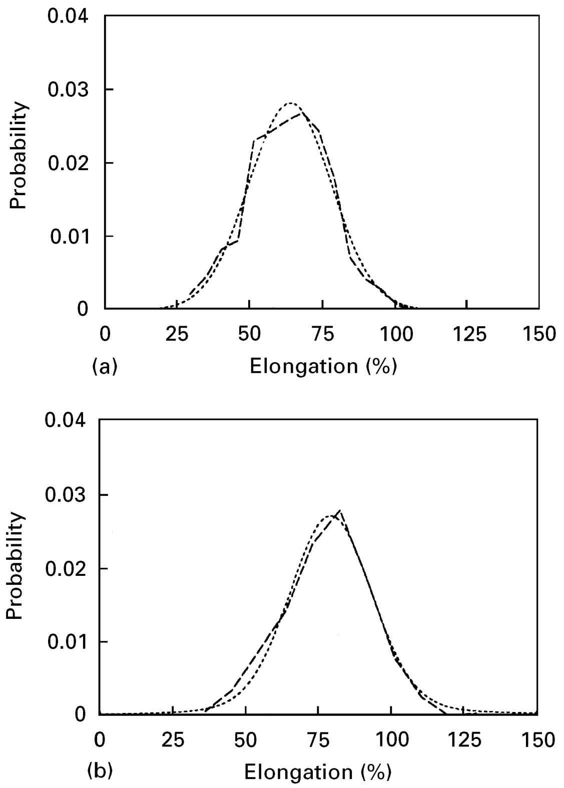

Figure 4 Experimental and the resolved elongation distributions of the polyester fibres: (a) polyester I and (b) polyester II.

TABLE IV Resolved parameters for the elongation distributions of the polyester fibres

Samples	f	A	W (%)	P (%)	χ^2
Polyester I	0.00	0.03	42.0	63.8	18.9
Polyester II	0.07	0.03	41.1	79.1	11.6

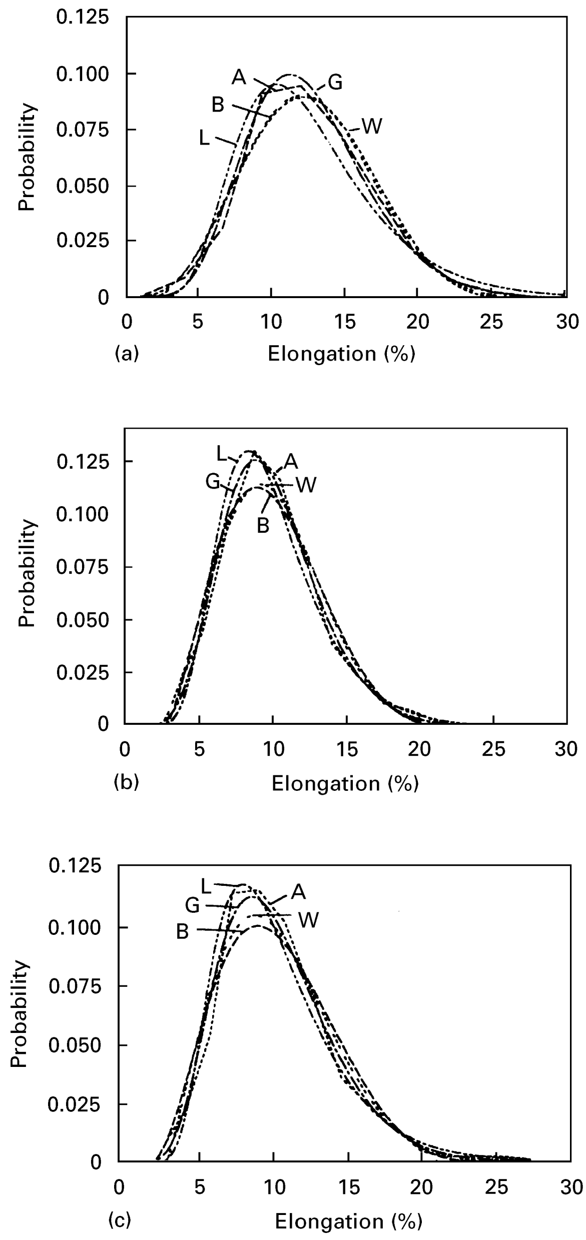

Figure 5 Experimental and the resolved elongation distributions with various standard statistical distribution functions for the cotton fibres (**A**, actual data; **W**, Weibull; **G**, Gamma; **B**, Beta; **L**, Lognormal): (a) cotton I, (b) cotton II and (c) cotton III.

The maximum amplitude position (P) of the elongation distribution is related to the expected value of the population. The two polyester fibres have elongations of 79.1% and 63.8%, respectively. The distribution

TABLE V Estimated parameters of the employed standard statistical functions

Distribution sample	Position c	Scale α	Shape m	Shape n	χ^2
Log-normal					
Cotton I	2.5		0.38		49.3
Cotton II	2.2		0.35		23.3
Cotton III	2.3		0.37		16.8
Rayon	3.2		0.26		181.1
Lyocell	3.0		0.19		42.0
Weibull					
Cotton I	2.0	813.6	2.71		30.7
Cotton II	2.3	176.5	2.41		31.6
Cotton III	2.8	100.3	2.14		18.2
Gamma					
Cotton I	0.1	0.72	9.0		18.2
Cotton II	2.6	0.90	8.9		13.7
Cotton III	1.6	0.74	7.6		9.3
Rayon	1.5	1.19	28.6		92.5
Lyocell	5.5	1.70	34.0		32.2
Beta					
Cotton I	0.1	28.9	4.65	6.11	35.9
Cotton II	2.6	21.1	2.84	5.31	55.0
Cotton III	1.6	24.6	3.18	5.71	24.7
Rayon	1.5	39.5	10.28	7.77	36.5
Lyocell	5.5	24.6	6.71	4.65	12.8

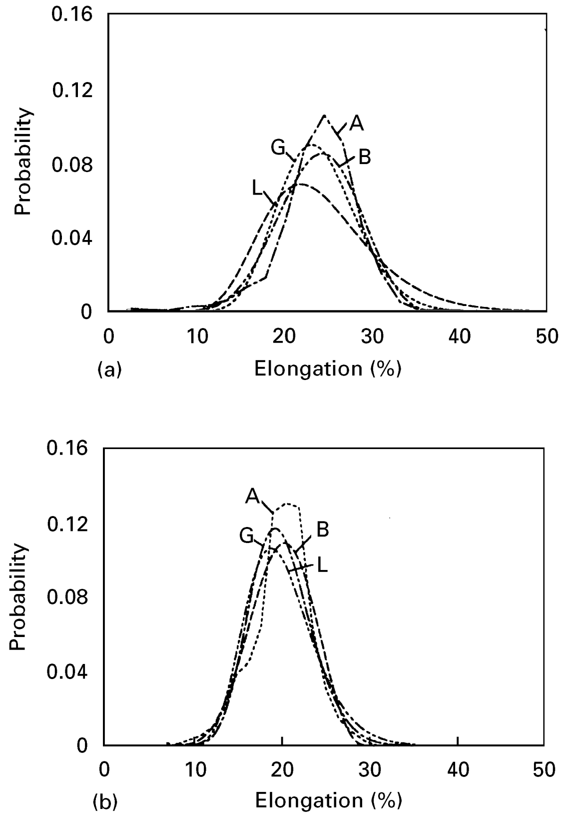

Figure 6 Experimental and the resolved elongation distributions with various standard statistical distribution functions for the regenerated cellulose fibres (**A**, actual data; **G**, Gamma; **B**, Beta, **L**, Lognormal): (a) rayon and (b) Lyocell.

width at the half maximum amplitude (W) is related to the variances of the population. The larger the distribution width, the larger the variance of the population. The distribution width of polyester fibres I and II are 41.6% and 38.6%, respectively, indicating large variances. The distribution maximum amplitude (A)

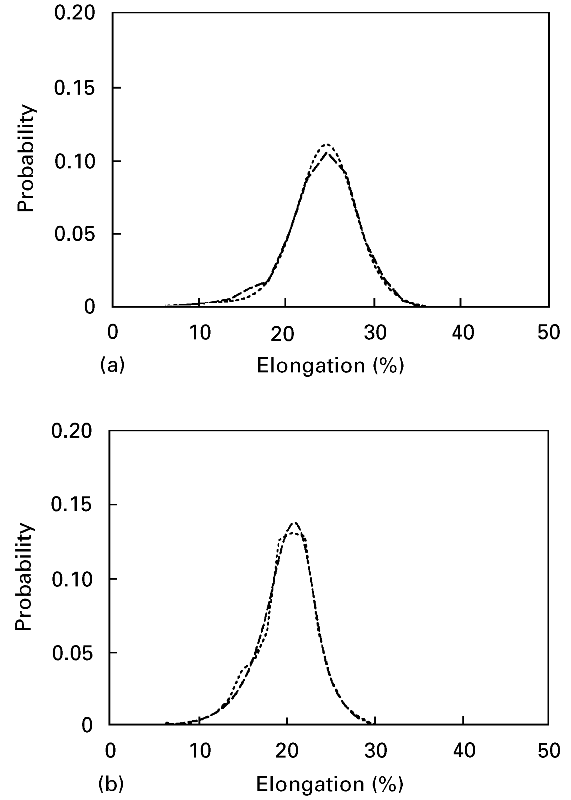

Figure 7 Experimental and the resolved elongation distributions with a mixture of normal and Gumbel-smallest functions for the regenerated cellulose fibres: (a) rayon and (b) Lyocell.

means the occurring probability of the mean value. Lower maximum amplitudes for the two polyester fibres (0.03) indicate lower occurring probabilities of the mean values.

3.3.2. Asymmetric distributions

Fig. 5 shows the curve-fittings of the breaking elongation distributions of cotton fibres using various standard statistical distribution functions. The corresponding parameters for the various functions were estimated by the maximum likelihood estimation method and are summarized in Table V. It has been found that the Weibull and Beta functions do not fit the data well near the top nor in the peak regions, whereas the Log-normal functions do not fit the data on the right side of the distribution. Therefore, our analyses of the breaking elongation distributions of cotton fibres also do not support the Rayleigh distribution [3], since the Rayleigh distribution is only a special case of Weibull distributions when the shape parameter m is equal to two.

The Gamma function seems to be the best curve-fitting function for all three cotton populations. The position parameter c has been set to be the minimum for each sample. For our cotton fibres, it is found that the shape parameter m is in the range 7.5 to 9.0 whereas the scale parameter α is in the range 0.7 to 0.9. The chi-square tests (χ^2) were employed for evaluating the goodness of fit (Table V). Under 95% confidence level, the chi-square results also show that the distributions of cotton fibres follow the Gamma distribution but not the other fibres.

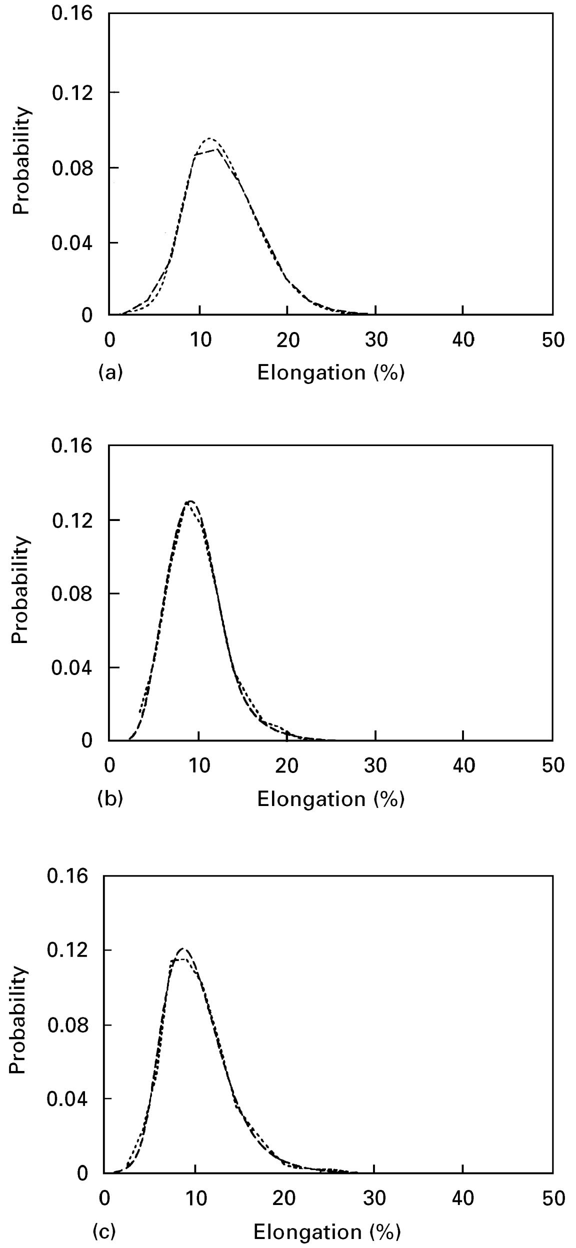

Figure 8 Experimental and the resolved elongation distributions with a mixture of normal and Gumbel-largest functions for the cotton fibres: (a) cotton I, (b) cotton II and (c) cotton III.

For the regenerated cellulose fibres with asymmetric distributions with negative skewness, none of the above standard statistical functions provide a good fit for the experimental data (Fig. 6 and Table V). A mixture of Gumbel (smallest) and normal functions shows much better curve-fittings for the rayon and Lyocell fibres (Fig. 7). The mixture of Gumbel (largest) and normal functions also give good fittings of the asymmetric distributions of the cotton fibres with positive skewness (Fig. 8). It is, therefore, concluded that a mixture of Gumbel and normal functions can provide reasonably good fit to any asymmetric distribution. The corresponding estimated parameters are calculated and summarized in Table VI. Again, by using the chi-square test, a mixture of Gumbel and normal functions is confirmed to provide the best fit for a fibre population with asymmetric distribution such as in the case of cotton and regenerated cellulose fibres.

TABLE VI Estimated parameters of the mixture of normal and Gumbel functions

Sample	μ_1	σ_1	μ_2	σ_2	f	χ^2
Smallest						
Rayon	24.4	3.17	25.7	6.94	0.20	10.4
Lyocell	22.9	2.62	20.5	2.48	0.75	11.0
Largest						
Cotton I	13.1	4.43	10.3	2.97	0.51	7.5
Cotton II	10.0	2.32	8.2	3.03	0.68	6.2
Cotton III	9.5	3.13	8.9	3.09	0.72	8.3

4. Conclusion

Single fibre breaking elongations of several fibres have been measured and their distributions analysed and reported. The elongation distributions of the three cotton fibre populations show positive skewness whereas those of the regenerated cellulose fibres show negative skewness. All have longer tails than normal distributions. Although being approximately symmetric, the elongation distributions of the two polyester fibres also show longer tails than normal distributions. Therefore, the elongation distributions of all these fibres can not be considered as pure normal distributions. For the polyester fibres with symmetric distributions, a mixture of Gaussian and Lorenzian functions can be employed to resolve their distributions. For rayon and Lyocell fibres with asymmetric distributions with negative skewness, none of the standard statistical distributions fit well. Therefore, a mixture of Gumbel (smallest) and normal functions is utilized to fit the elongation distributions. For the cotton fibres, which have asymmetric distributions with positive skewness, the Gamma function seems to provide the best-fit when comparisons are made with several other statistical distributions (Beta, Weibull, Log-normal). Additionally, a mixture of Gumbel (largest) and normal functions can also fit these asymmetric distributions with positive skewness. The statistical approaches demonstrated here are excellent tools for describing the distribution function of single fibre tensile properties. Understanding the distribution functions of single fibre properties can help to improve the properties of fibrous assemblies through optimal fibre mix.

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